Key

Math 4

~~U7 L14~~ 5-5 **Practice**

Name _____

Date _____

1. A particle moves along the x-axis in such a way that its position at time "t" is given by $s(t) = 3t^4 - 16t^3 + 24t^2$ for $-5 \le t < 5$.

   a. Determine the velocity and acceleration of the particle at time "t". [find $v(t)$ and $a(t)$.]

   b. For what values of t is the particle at rest?

   c. For what values of t does the particle change direction?

   d. What is the velocity when the acceleration is first zero?

   e. When is the particle speeding up? When is it slowing down?

a) $v(t) = 12t^3 - 48t^2 + 48t$ $\quad a(t) = 36t^2 - 96t + 48$

b) $v(t) = 0$ ... $0 = 12t(t^2 - 4t + 4)$ $\quad 0 = 12t(t-2)(t-2)$ $\quad \boxed{t = 0, 2 \text{ sec}}$

c) $0 = 12t(t-2)(t-2)$

$t = 0, 2$

	$t<0$	$0<t<2$	$t>2$
$12t$	$-$	$+$	$+$
$t-2$	$-$	$-$	$+$
$t-2$	$-$	$-$	$+$
	Left	Right	Right

Change in direction

Changes direction when $t = 0$.

d) $0 = 36t^2 - 96t + 48$

$0 = 12(3t^2 - 8t + 4)$

$0 = 12(3t-2)(t-2)$

$t = \frac{2}{3}$ $\quad t = 2$

Acceleration first zero when $t = \frac{2}{3}$ sec.

$v\left(\frac{2}{3}\right) = 12\left(\frac{2}{3}\right)^3 - 48\left(\frac{2}{3}\right)^2 + 48\left(\frac{2}{3}\right)$

$= \boxed{\frac{128}{9} = 14.\overline{2} \text{ ft/s}}$

e) $0 = 12(3t-2)(t-2)$

$t = \frac{2}{3}, t = 2$

	$t<\frac{2}{3}$	$\frac{2}{3}<t<2$	$t>2$
$3t-2$	$-$	$+$	$+$
$t-2$	$-$	$-$	$+$
	$+$	$-$	$+$
	speed up	slow down	speed up

Speed up: ~~$t < \frac{2}{3}, t > 2$~~

V & a opp. signs

Slow down: ~~$\frac{2}{3} < t < 2$~~ $t < 2$

~~$0 \le t \le 5$~~ $2 < t < 5$

V & a both + or −

2. A projectile is fired straight upward with a velocity of 400 ft/sec. Its distance above the ground t seconds after being fired is given by $s(t) = -16t^2 + 400t$

   a. Find the time and the velocity at which the projectile hits the ground.

   b. What is its maximum altitude?

   c. What is the acceleration at any time t?

a) Position $= 0$

$0 = -16t^2 + 400t$

$0 = -16t(t - 25)$

$t = 0$ $\quad \boxed{t = 25}$

$t = 0$: Start $\quad t = 25$: Back to ground

$v(t) = -32t + 400$

$\boxed{v(25) = -400 \text{ ft/s}}$

b) Reaches max height when $v(t) = 0$

$0 = -32t + 400$

$\boxed{t = 12.5 \text{ seconds}}$ ← not answer

Answer = $\boxed{2{,}500 \text{ ft}}$

c) $\boxed{a(t) = -32 \text{ ft/s}^2}$

3. A particle moves along the x-axis in such a way that its position at time t is given by

$$s(t)=\frac{1}{3}t^3-3t^2+8t,\ (t>0)$$

a. Show that at time $t=0$ the particle is moving to the right.

b. Find all values of t for which the particle is moving to the left.

c. What is the position of the particle at time $t=3$?

d. When $t=3$, what is the total distance the particle has traveled?

a) $v(t)=t^2-6t+8$

$v(0)=0^2-6(0)+8$

$=8$ → positive = moving right

b) $0=t^2-6t+8$

$0=(t-4)(t-2)$

$t=4 \quad t=2$ → $2<t<4$

	$-\infty$		2		4		∞
$t-4$	−		−		+		
$t-2$	−		+		+		

+ Right 2 Left 4 + Right

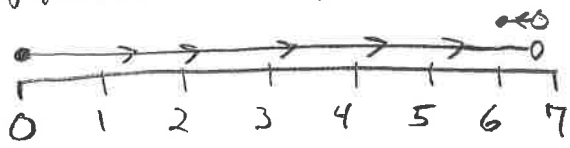
c) $s(3)=$ 6 units

d) Started at 0 units − $s(0)=0$

$s(2)=6\frac{2}{3}$ units

$s(3)=6$ units

*** Visualize the particle on the x-axis !!



$6\frac{2}{3}$ right + $\frac{2}{3}$ left

$= 7\frac{1}{3}$ units

4. The position of a particle in motion is measured by its distance away from its starting point. For $0\le t\le 4$, the particle's position (in feet) over time (in seconds) is modeled by the function $s(t)=\frac{1}{3}t^3-\frac{5}{2}t^2+4t$

a. What function models the velocity of the particle

b. What function models the acceleration of the particle?

c. At what time(s) is the particle at rest?

d. What is the particle's velocity at 3 second?

e. What is the particle's acceleration at 3 second?

f. At $t=3$, is the particle speeding up or slowing down? Show your work and explain.

a) $s'(t)=v(t)=t^2-5t+4$

b) $s''(t)=a(t)=2t-5$

c) $0=v(t)$

$0=t^2-5t+4$

$0=(t-4)(t-1)$

$t=4$ sec $\quad t=1$ sec

d) $v(3)=3^2-5(3)+4$

$=-2$ ft/s

e) $a(3)=2(3)-5$

$=1$ ft/s^2

f) $v(3)=-2$ ft/s

$a(3)=1$ ft/s

Velocity is negative + acceleration is positive, so the particle is slowing down.